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THE PROVISIONAL REPORT OF THE NATIONAL COMMITTEE ON A GEOMETRY SYLLABUS.

By Howard F. Hart.

We have before us for discussion the Provisional Report of the National Committee of Fifteen on a Geometry Syllabus, The history of the formation of this committee and the results of its subcommittee on Logical Considerations have already been presented to this Association by Mr. E. R. Smith and printed in the Mathematics Teacher for June, 1910. When it was suggested to me that I discuss this report before you I hesitated because the form of this report, in which at times it appears to be the habit of the committee to propose a change, then debate the question and finally straddle the fence, does not lend itself to criticism, and also because in general this is a good report and the result of much work. I accepted in the hope that certain features of this report might be discussed and the Association take definite action with respect to them.

The report is divided into five sections as follows: (1) An historical summary of attempts to change the geometry course in France, Germany, Italy, England, and the United States, (2) Logical considerations, (3) Special courses, (4) Exercises, (5) Syllabus of propositions. I propose to omit any consideration of the first and third sections and only to make such references to the second section as are necessary. This should give us time for consideration of the larger things advocated by the committee. These I take to be:

I. The claims for geometry.

II. The attitude toward definitions.

III. The recommendation of informal proofs for some propositions now formally proved.

IV. The position with respect to the incommensurable case.

V. The recommendation on the distribution of exercises.

VI. The recommendation on the nature of exercises.

VII. The use of the concept of motion.

VIII. The use of algebraic notation in proofs.

IX. The geometrical construction of formulas.

X. The recommendation of the introduction of the trigonometric ratios.

XI. The syllabus proper.

After a warning against the dangers of dull formalism on the one hand and extreme radicalism in applications on the other the committee contends for geometry that it is primarily an exercise in logic and in such unusually simple setting that its results can easily be carried over into the lives of the students. In addition to the logical part is the training gained from accurate and precise thinking and the expression of such thinking together with the experience gained by contact with exact truth. Geometry also develops the idea of functionality, the power of space intuition and visualization, and is of course the basis of mensuration. In all these things geometry is an unique study in the secondary curriculum. We should all be glad that the committee obviously felt that there was no occasion or need for any excuses of any sort. To quote directly, "If we had to justify the position of any other subject in the curriculum history, rhetoric, geography, biology, etc., it is doubtful whether equally specific and cogent reasons could be found."

In the matter of definitions the recommendations of the committee can only lead to much good. In the first place it is recognized that certain terms cannot be defined in the strictly logical sense. In such cases the committee recommends that no so-called definition be attempted but that the concept be most carefully built up; but where definitions are possible and proofs depend upon them the definitions should be given. Also the committee is not in favor of definitions being taught other than at the point where they are needed. It regards the form of a definition as immaterial so long as it is as scientific as it should be for a secondary school.

The committee feels that it is not worth while to prove formally certain types of propositions. It gives among other examples, (1) All straight angles as equal, (2) Two straight lines can meet but once, (3) Polygons similar to the same polygon are similar to each other, (4) A straight line can cut a circle at most in two points. The committee admit the dangers involved in the carrying of such a thing beyond reasonable limits and close their debate on the point as follows: "With all the experiments at improving Euclid the world has really accomplished very little except as to the phraseology of propositions and proofs; the standard propositions remain, and if geometry has any justification, apart from its kindergarten aspect (which requires but a short time), most of these propositions will continue to be proved, and should continue to be proved." It is obvious that a proposition which is an immediate influence from a definition can be best so derived on the spot, *e. g.*, in the second illustration it would probably be done somewhat like this: Two straight lines can meet but once, for if they were to meet twice they would coincide (straight line axiom) but since they are not coincident (given) they meet but once (negative-converse law). I assume that this is the kind of thing recommended and on that basis commend it.

Limits and incommensurable cases have long been a source of trouble. Constant, variable, limit, can be easily enough developed as concepts but the use of the Theory of Limits to prove incommensurable cases does not seem to be equally developable. So the committee recommended that commensurable cases only be required by any examining body. The present tacit understanding is unfortunate in that examiners cannot set a proposition which has the two parts. If this recommendation is definitely adopted it will end an uncertainty and be a relief to everybody.

In writing proofs the use of the symbolic or algebraic notation is advocated. It is claimed that the dangers will be obviated if the students be required to give the full equivalent in words in all oral work. My own experience, and I have been using an operative system of symbols in solid geometry for some years now (this experience was kindly printed by the MATHEMATICS TEACHER for September, 1910) makes me feel that the recommendation is very valuable and that the dangers can be obviated and no bad effects need result. The dangers involved are, however, very real ones. For an illustration let us consider a right-angled triangle. There are two propositions based on this figure which have to be carefully differentiated. They are (1) "The square on the hypothenuse equals the sum of the squares on the legs," (2) "The square of (the length of) the hypothenuse equals the sum of the squares of (the lengths of) the legs." The tendency would be to make one symbol a^2 represent two such different things as a limited portion of a plane and a number; and, unless the teacher is everlastingly vigilant, that is just what will happen. In fact it happens in this very syllabus.

The committee recommended that in developing loci theorems the phraseology used be descriptive of a point moving under limiting conditions rather than of an assemblage of fixed points each satisfying the conditions. In my opinion there is no demand for this and I see no great advantage that would result from the change. For we all know that the point, which in a loci theorem we are either proving to satisfy the condition because it is on the line or conversely, is a fixed point. That it was moving before or will move later is not essential to the proof The committee base their recommendation on the fact that the student is already pretty familiar with motion as a concrete experience and that we should therefore use this experience. I doubt if it would prove very helpful. As in the well-known case of measurement, with which also the student is concretely familiar but thinkingly almost an entire stranger, so it will be here. The common experience of beginners in analytical geometry is further testimony in the same direction.

On the other hand the recommendation that the natural trigonometric functions of acute angles be introduced is at least feasible. That the students should, however, compute, or determine by approximate measurement, a two-place table of functions is not equally worth while in my opinion. On its pure mathematics side there is little (if anything) to be said for it. As an application it would be a good laboratory exercise in natural science. We must never forget that geometry is about the only course in the secondary curriculum which is ideal in the philosophic sense, nor must we ever adopt any procedure which will give it the air of a laboratory course in physical quantities. Personally I am opposed to having my students make their construction work an approximation process. As an illustration: "Given the line AB and an external point C to draw a perpendicular through C to AB." In this construction I have uniformly rejected any solution given in which the radius of the circle to be drawn, with C as a center, is not properly determined. Finally, in very few schools indeed will there even be time for the introduction and proper use of the ratios only.

The recommendation that part of the exercises be placed under the theorems to which they are corollaries instead of all the exercises being grouped at the end of a section, must commend itself to all. This is current practice in the best books.

The recommendations on the nature of the exercises are to quote directly, (1) "There are several types of genuine problems, but many of the so-called real applications either are too technical to be within the grasp of the young beginner or represent methods of procedure that would not be followed in real life. Moreover, it should be remembered that the very limited time devoted to plane geometry renders it impracticable to introduce many of the applications that might be desirable if the time were not so restricted." Later it is recommended, "that a judicious selection of a reasonable number of abstract originals be made in order to leave time for an equally reasonable number of problems, particularly those with local coloring, stated in concrete setting." If the committee means what it says in both the quotations then it may be a consistent recommendation and descriptive of an ideal. But I think it only fair to say that pages forty-seven to fifty-four immediately following the second quotation, in which are set forth in detail sources of problems from architecture, decoration, design, together with a bibliography of the "real problem" propaganda would make one think that the committee actually recommends a maximum of "problems in concrete setting." I hope that on this one point at least the Association will take definite action.

I am in hearty accord with the suggestion that the geometric construction of equivalence formulæ be a part of a standard course.

We come now to the syllabus proper. Probably the first thing that will be noted is the way the theorems are printed. This was the means adopted to indicate the relative importance of the theorems. Those in heavy type are basal theorems of their group; those in italics are of considerable importance, but secondary to the first; those in roman type are still less important; while those in small type could be "omitted without serious danger." This is in my opinion a fundamentally helpful and necessary improvement. An attempt to realize this is made by all good teachers. For another means to get this result compare Professor Peck's article (p. 54) in the MATHEMATICS TEACHER for December, 1910.

When we come to study the working out of this idea in this syllabus we meet some radically new points of view on certain theorems. Perhaps the most radical one is the reduction of inequality propositions to that class which can be "omitted without serious danger." Thus (1) If two sides of a triangle are unequal the opposite angles are unequal and the greatest angle is opposite the greatest side and conversely; (2) The exterior angle is greater than either interior angle not adjacent; (3) In two triangles having two sides respectively equal if the included angles are unequal the third sides are respectively unequal in the same order and conversely, (4) Of all the lines drawn from a point to a line, etc.; (5) In the same or equal circles the greater of two chords is nearer the center and conversely, etc., can be "omitted without serious danger." I am wondering why the committee adopted this. Was it to shorten the list some way, some how, so as to give time for "problems in concrete setting"? Or was it because the proof of the inequality propositions generally involve the proof of a location? If the latter case, are we to understand that the committee would only have us study such configurations as present no serious difficulties? A further study of this question, particularly in view of the theorems for solid geometry, makes us wonder still more, for there the inequality propositions are not printed in small type and cannot, as we know well, be omitted. Yet their proof requires the inequality theorems of plane geometry.

Continuing our study of small type theorems, we observe that the committee is willing to have omitted (1) the theorem which tells us when two circles intersect and is therefore the basis of constructions, (2) the construction of a tangent, (3) all propositions on the concurrent lines of a triangle, (4) the ways of proving lines not parallel by means of angles, (5) all propositions on the locus of the vertex of a fixed angle opposite a fixed line segment though this is one of the two great ways of realizing circle loci, etc., etc. The following are absolutely omitted, no reference being made to them in any way: (1) The theorems of proportion as such; (2) lines perpendicular to intersecting lines; (3) the sum of the exterior angles of any convex polygon; (4) the line joining the midpoints of the sides of a triangle (though the general proposition is given, X. 3); (5) the midpoint of the hypothenuse of a straight triangle; (6) the inscriptible quadilateral; (7) the development of arcs from proportion (to be assumed as definition); (8) construction of a triangle equivalent to a given polygon, etc.; (9) harmonic division of a line segment, etc., etc.

In the syllabus of propositions for solid geometry we find among the theorems for informal proof the one on the intersection of two planes though this is an important locus theorem and should be proved as such. I assumed that the new axiom, "Two intersecting planes have at least two points in common," was given for that very purpose. So also we find here the proposition on the measure of the dihedral angle. From the point of view of its importance it should be in as heavy type as any proposition in the syllabus. And furthermore, it should be divided into the three customary propositions and proved in that way.

In the next section the committee takes the position that "Two straight lines are parallel to each other *if and only if* they are both perpendicular to some one plane." I might say that three planes in space can intersect in three lines and that, if they do, these three lines must either be concurrent or parallel. This can be demonstrated without any reference to being perpendicular to any plane whatsoever. And it is a sufficient basis for showing that lines parallel to the same line are parallel to each other.

I might continue this line of criticism to much greater length but will stop with the observation that this syllabus will be famous, or otherwise, for what it omits.

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